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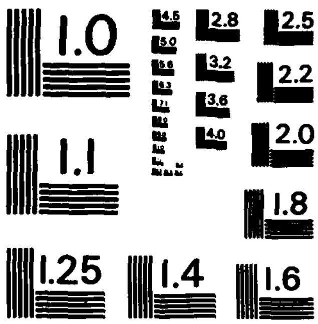

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Aerodynamics Technical Memorandum 366

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Approved for public release.

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CALCULATION OF THE VELOCITY FIELD GENERATED BY A HELICOPTER MAIN AND TAIL ROTORS IN HOVER

by

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SUMMARY

A theory is presented to calculate the flow field due to main and tail rotors of a helicopter in hovering flight. To solve this complex flow problem, simple vortex rings are used to model main and tail rotor wakes. Preliminary numerical results are found to be encouraging.

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CONTENTS

NOTATION

a,b,c	co-ordinates of the centre of the tail rotor
C_T	rotor thrust coefficient
h	vertical distance between field point Q and the vortex ring
k	vortex strength of a ring vortex
r,θ	radial and angular co-ordinates (see fig. 1)
R	radius of vortex ring
s	ring vortex circumferencial parameter
$\bar{t}$	local tangent vector
$\bar{u}$	velocity vector in cylindrical polar co-ordinates
u_r,u_z	radial and axial velocity components
v_i	induced velocity in the plane of the rotor
$\bar{x}$	position vector of field point Q (see fig. 1)
$\hat{x},\hat{y},\hat{z}$	unit vector in the Cartesian co-ordinate system x,y,z (see fig. 1)
ϕ	azimuthal position (see fig. 1)

1. INTRODUCTION

In certain helicopter flight conditions, the fuselage, and other components such as the tail rotor, fin and hub, are partially or wholly immersed in the main rotor slipstream. This leads to interactional aerodynamic phenomena which have been observed in many helicopters [1]. A clear knowledge of these phenomena is important not only because they could be a serious source of vibration and fatigue [2], but also because they may affect the aerodynamic efficiency of various components. The present trend towards greater compactness necessitates the various aerodynamic elements being close to each other. This means that the aerodynamic interaction problem will become even more acute in the future.

Because of the large number of helicopter aerodynamic components involved, a comprehensive mathematical analysis of the interaction phenomena is extremely difficult. Mutual interaction of the vortex elements generated by the separate components will deform the vortices which, because of viscosity, will eventually dissipate. To solve this complex interactive flow problem, various simplifying assumptions are necessary. In particular, these include the number and geometry of the aerodynamic elements, and the flow field generated by each element. In this document only the two important aerodynamic elements, the main rotor and tail rotor, are considered.

The wake model to represent a helicopter rotor blade varies from very complex lifting line theory to simple blade tip vortex theory. As an initial step, a simple tip vortex wake model is used here to represent the wake of a rotor blade. To simplify the numerical process further, the trailing vortex pattern is assumed to form a series of vortex rings. Similar wake models have been used previously by Cook [3] to calculate the helicopter main rotor performance, and recently Rayner [4] used a ring vortex wake model to study hovering flight of animals. Other forms of representing the wake such as the skewed spiral, necessitate numerical integration and are therefore time consuming.

In developing the theory, consideration has been limited to aircraft having two rotors, although no restriction has been placed on the number of rotor blades. A simple and efficient procedure, suitable for flight simulation studies, for the calculation of the flow field due to main and tail rotors is described. For the test case, the induced flow in the plane of the main rotor in steady hovering flight has been calculated and compared with results from a more complex model. The theory can be readily extended to cope with forward flight. Throughout, it is assumed that the fluid is inviscid and incompressible.

2. DEVELOPMENT OF THE WAKE MODEL

To estimate the combined main and tail rotor velocity field, the wake of each rotor is modelled as a stack of circular coaxial vortex rings beneath the rotor. The rings are equally spaced,

the first one being half the spacing distance below the rotor. The momentum theory is used to calculate the spacing between the rings.

Initially, all the rings were the same radius as the rotor so as to simplify the situation but a comparison with results from other models (Reference 5) showed a large variation in results. The radii of the rings are now given by -

$$R = (0.78 + 0.22\ e^{-K\psi})\ R'$$

(see Reference 6) where K is a wake contraction parameter ($=4(C_T)^{\frac{1}{2}}$), $\psi = 2\pi(i-1)$ for the ith vortex ring and R' is the radius of the rotor. This gives much improved results for induced velocity in the plane of the rotor, except for $r/R' > 0.975$ - the very edge of the rotor disc.

The vortex rings are assumed to be sufficiently far apart so that they have no effect on one another. Thus, the total velocity induced by the entire "stack" of rings is just the vector sum of the velocities induced by the individual rings.

The tail rotor is modelled in exactly the same way as the main rotor, although a different co-ordinate system is used, and the results can then be combined with those from the main rotor to give the total induced velocity. For the data used, however, the contribution to the induced velocity from the tail rotor is small except near the rear edge of the main rotor disc.

3. DESCRIPTION OF THE COMPUTER PROGRAM

The program itself is a straight-forward implementation of the formulae developed in the Appendix. The required inputs are:-

- the radius of the rotor
- the separation between consecutive vortex rings
- the number of vortex rings to consider
- the circulation of each ring.

If the effects of a tail rotor are to be included then this information is also needed for the tail rotor, as well as the position of the centre of the tail rotor relative to the main rotor.

The induced velocity at the rotor is then found using the information provided, by application of the formulae in the Appendix. If the tail rotor is included, the induced velocity due to each stack of rings is found and the results combined to give the total effect. In this case, the effect of each rotor is output, as well as the total effect.

The units of the output are determined by those of the input data. If distances (i.e. radii and separations) are given in metres and circulations in metres^2/second, then the results are in m/s, but if other output units are desired then the input data must be modified appropriately.

The induced velocities are calculated using two right-handed Cartesian coordinate systems. The main one, to which all input and output is referenced, is centred at the centre of the main rotor, has the x-axis directed horizontally backwards, the y-axis horizontal and the z-axis vertically upwards. The second system, which is used for convenience in calculating induced velocities due to the tail rotor, is parallel to but rotated from the main system as shown in Figure 2.

If the centre of the second co-ordinate system is (a,b,c) relative to the main system, then the point P(x,y,z) in the main system is given by P'(x-a,z-c, b-y) relative to the second system.

The program was written in modular form, so that the main program controls the flow of execution but does very little of the actual work; most of the work is done in subroutines to read in the input values and calculate the induced velocities by application of the formulae. Note that the calculations require the use of a numerical integration routine from the library DSKB.NAGF [7,7], which must be included at link time for the program to run correctly.

4. RESULTS AND CONCLUDING REMARKS

The wake in the hovering flight of a rotor is modelled by a stack of circular blade tip vortex rings. Experimental evidence suggests that blade tip vortex is the most dominent element in the rotor wake. Experimental results also show that this blade tip vortex though highly distorted farther away from the rotor, follows a path quite close to a circle in the immediate vicinity of the rotor. If calculation of the velocity field close to the rotor is of main concern then representation of the rotor vortex wake by a stack of vortex rings is physically reasonable. This also simplifies the mathematical analysis, the computer time is reduced which make the model very suitable for flight simulation studies.

Induced velocities in the rotor plane calculated using the ring vortex wake modelare presented in Figure 3 for a typical four bladed helicopter rotor in hover. A comparison of results from this model with those in Reference 5 shows reasonable agreement between the theories except at the edge of the blade, where some of the assumptions made deriving the current formulae may not hold. Methods to improve the present mathematical model are being investigated. The theory can be readily extended to cover forward flight of any two rotor helicopter.

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APPENDIX

MATHEMATICAL DERIVATION OF THE VELOCITY FIELD INDUCED BY A VORTEX RING ELEMENT (from reference 4)

Consider the effect of a single vortex ring, centred at the origin and lying in the x-y plane, on a point in the x-z plane (fig. 1a). No generality is lost by this positioning of the point as any point can be so located by a rotation of the co-ordinate system about the z-axis.

As long as the point is not near the core of a vortex ring the vortex core can be approximated by a line vortex, and the induced velocity field is given by the Biot-Savart law.

$$\bar{u}(\bar{x}) = \frac{-k}{4\pi} \oint \frac{(\bar{x}-\bar{X}(s))\wedge\bar{t}(s)}{||\bar{x}-\bar{X}(s)||^3}\, ds \qquad (1)$$

where s is a parameter of the circumference of the vortex ring.

In the case of figure 1a with points P and Q (assumed to be in the x-z plane),

$$\begin{aligned} \bar{X} &= R(\cos\theta,\ \sin\theta,\ 0) \\ \bar{x} &= (r,o,h) \\ s &= R\theta \\ \bar{t} &= \frac{d\bar{X}}{ds} = (-\sin\theta,\ \cos\theta,\ 0) \end{aligned} \qquad (2)$$

so from (1) it follows that

$$\bar{u}(\bar{x}) = \frac{kR}{4\pi} \int_0^{2\pi} \frac{(h\cos\theta,\ h\sin\theta,\ R-r\cos\theta)}{[r^2-2rR\cos\theta + R^2 + h^2]^{3/2}}\, d\theta \qquad (3)$$

Denote the unit vectors in the x,y and z directions by $\hat{x},\hat{y},\hat{z}$ respectively.

By symmetry about the z-axis

$$\bar{u}.\hat{y} = 0 \tag{4}$$

Define

$$\bar{u}.\hat{x} = \frac{kRh}{4\pi} I_2(r,R,h)$$

$$\bar{u}.\hat{z} = \frac{kR}{4\pi} (RI_1(r,R,h) - rI_2(r,R,h)) \tag{5}$$

where I_1 and I_2 can be evaluated in terms of the complete elliptic integrals

$$K(e) = \int_0^{\pi/2} (1-e^2\cos^2\bar{\theta})^{-\frac{1}{2}}\, d\bar{\theta}$$

(6)

and $$E(e) = \int_0^{\pi/2} (1-e^2\cos^2\bar{\theta})^{\frac{1}{2}}\, d\bar{\theta}$$

where the eccentricity e is given by

$$e^2 = 4rR[(r+R)^2 + h^2]^{-1} \tag{7}$$

For K(e) to be real, $e^2 \leq 1$ which is always true as follows

$$(r-R)^2 + h^2 \geq 0 \qquad \forall\, r,R,h$$

$$\Rightarrow \quad r^2 + 2rR + R^2 + h^2 \geq 4rR$$

$$\Rightarrow \quad (r+R)^2 + h^2 \geq 4rR$$

$$\Rightarrow \quad \frac{4rR}{[(r+R)^2 + h^2]} = e^2 \leq 1$$

Comparing (3) and (5) gives

$$\bar{u}.\hat{x} = \frac{kRh}{4\pi} \int_0^{2\pi} \frac{\cos\theta\, d\theta}{[(r+R)^2+h^2-2rR-2rR\cos\theta]^{3/2}}$$

(8)

$$= \frac{kRh}{4\pi} I_2(r,R,h)$$

$$\therefore\ I_2 = \int_0^{2\pi} \cos\theta[(r+R)^2+h^2-2rR(1+\cos\theta)]^{-3/2}\, d\theta \qquad (9)$$

$$\therefore\ I_2 = [(r+R)^2+h^2]^{-3/2} \int_0^{2\pi} \cos\theta \left[1-e^2\frac{(1+\cos\theta)}{2}\right]^{-3/2} d\theta \qquad (10)$$

But $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$\therefore\ I_2 = 2[(r+R)^2+h^2]^{-3/2} \int_0^{\pi} \cos 2\bar{\theta}[1-e^2\cos^2\bar{\theta}]^{-3/2}\, d\bar{\theta} \qquad (11)$$

Also note

$$\int_0^{\pi} \cos 2\bar{\theta}[1-e^2\cos^2\bar{\theta}]^{-3/2}\, d\bar{\theta} = 2\int_0^{\pi/2} \cos 2\bar{\theta}[1-e^2\cos^2\bar{\theta}]^{-3/2}\, d\bar{\theta} \qquad (12)$$

Thus

$$I_2 = 4[(r+R)^2+h^2]^{-3/2} \int_0^{\pi/2} \cos 2\bar{\theta}[1-e^2\cos^2\bar{\theta}]^{-3/2}\, d\bar{\theta} \qquad (13)$$

$$= 4H(e)[(r+R)^2+h^2]^{-3/2} \qquad (14)$$

from tables of integrals, where H(e) is defined by

$$H(e) = \frac{1}{e^2}\left[\frac{2-e^2}{1-e^2}\, E(e) - 2K(e)\right] \qquad (15)$$

Substituting (9) back into (5) and comparing the result to (3) and proceeding similarly to the above gives

$$I_1 = 4[(r+R)^2+h^2]^{-3/2} \int_0^{\pi/2} (1-e^2\cos^2\bar{\theta})^{-3/2}\, d\bar{\theta} \qquad (16)$$

$$= 4G(e)\ [(r+R)^2+h^2]^{-3/2} \qquad (17)$$

where $G(e) = \frac{E(e)}{(1-e^2)}$ (18)

From (14) and (17) it can be seen that

$$\bar{u} = u_r \hat{x} + u_z \hat{z} \tag{19}$$

$$u_r = \frac{kRh}{4\pi} \frac{4}{[(r+R)^2+h^2]^{3/2}} H(e) \tag{20}$$

$$u_z = \frac{kR}{4\pi} \frac{4}{[(r+R)^2+h^2]^{3/2}} (RG(e)-rH(e)) \tag{21}$$

with e given as in (7).

If the point Q does not lie in the x-z plane then the quantity u_r is a radial velocity directed outwards from the z-axis, which must be split into true x and y components.

Then

$$\begin{aligned} u \hat{x} &= u_r \cos \phi \\ u \hat{y} &= u_r \sin \phi \end{aligned} \tag{22}$$

where ϕ is the angle shown in figure 1b.

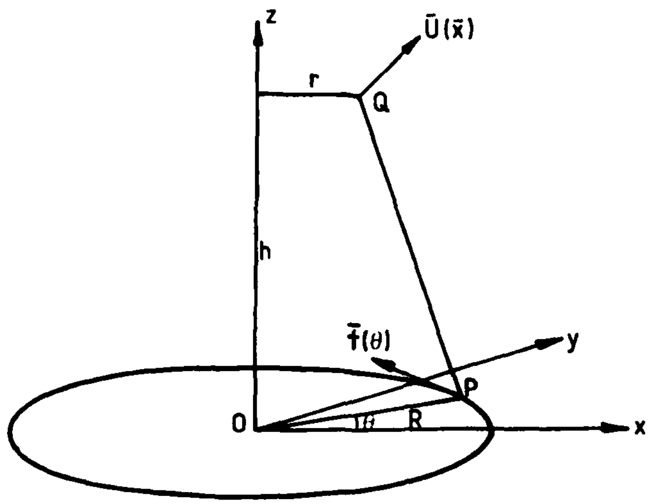

FIG. 1(a) FLOW INDUCED BY A VORTEX RING.

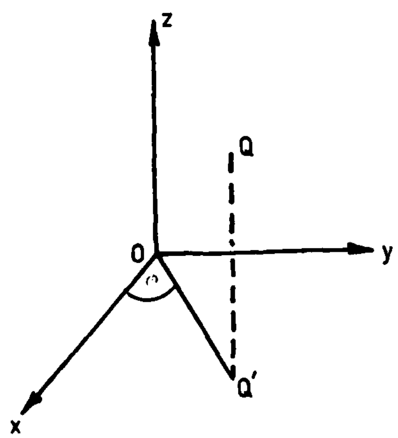

FIG. 1(b) RESOLVING THE VELOCITY COMPONENTS WHEN POINT Q DOES NOT LIE IN THE x-z PLANE.

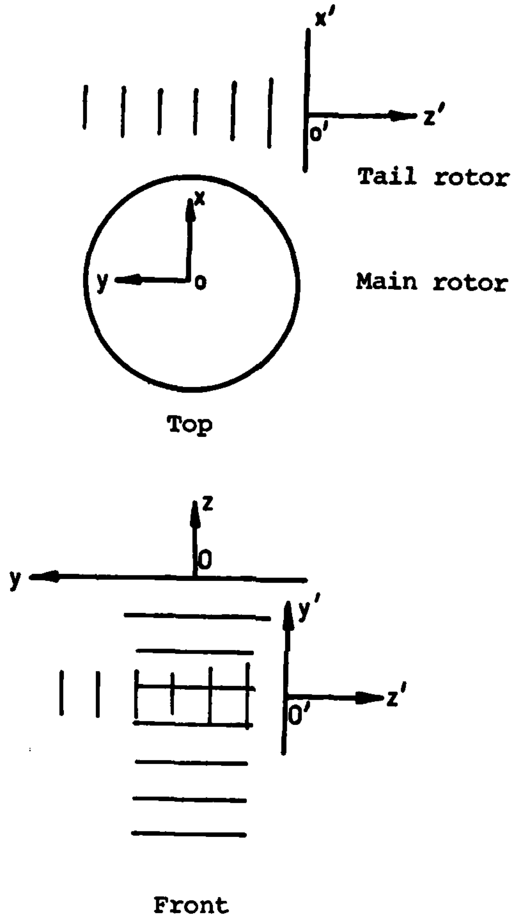

FIG. 2 CO-ORDINATE SYSTEMS

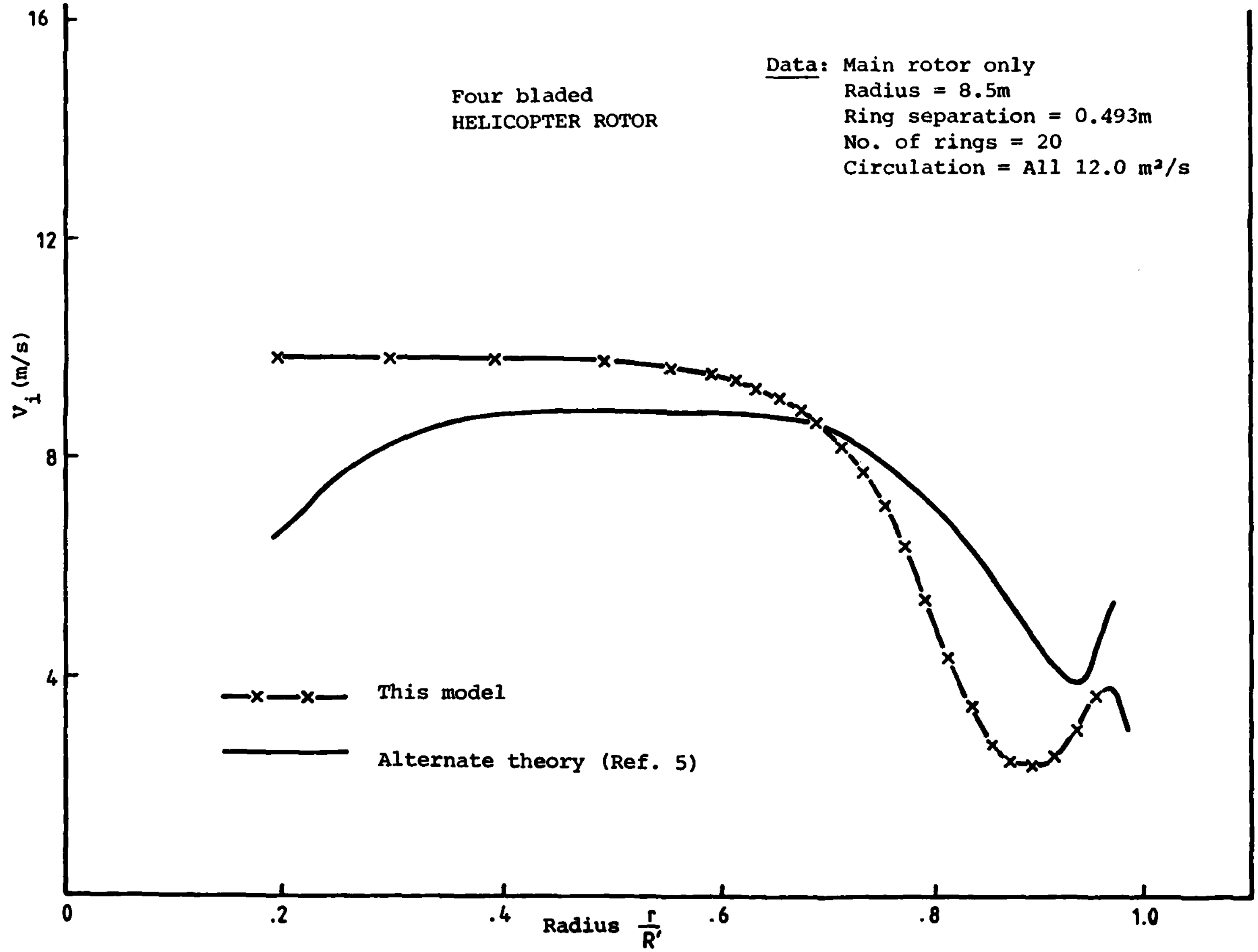

FIG. 3 VARIATION OF INDUCED VELOCITY WITH ROTOR RADIUS IN THE PLANE OF THE ROTOR.

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